Enabling Efficient and General Subpopulation Analytics in Multidimensional Data Streams

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ABSTRACT

Today’s large-scale services (e.g., video streaming platforms, data centers, sensor grids) need diverse real-time summary statistics across multiple subpopulations of multidimensional datasets. However, state-of-the-art frameworks do not offer general and accurate analytics in real time at reasonable costs. The root cause is the combinatorial explosion of data subpopulations and the diversity of summary statistics we need to monitor simultaneously. We present Hydra, an efficient framework for multidimensional analytics that presents a novel combination of using a “sketch of sketches” to avoid the overhead of monitoring exponentially-many subpopulations and universal sketching to ensure accurate estimates for multiple statistics. We build Hydra as an Apache Spark plugin and address practical system challenges to minimize overheads at scale. Across multiple real-world and synthetic multidimensional datasets, we show that Hydra can achieve robust error bounds and is an order of magnitude more efficient in terms of operational cost and memory footprint than existing frameworks (e.g., Spark, Druid) while ensuring interactive estimation times.

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The source code, data, and/or other artifacts have been made available at https://github.com/antonis-m/HYDRA_VLDB.

1 INTRODUCTION

Many large-scale infrastructures (e.g., Internet services, sensor farms, datacenter monitoring) produce multidimensional data streams that are growing both in data volume and dimensionality [2, 25, 79]. These multidimensional data contain measurements of metrics along with metadata that describe said measurements across domain-specific dimensions. For instance, video streaming services analyze user experience issues across dimensions, such as ISP, CDN, Device, City, etc. [60, 61]. We see similar trends in other domains e.g., network and data center monitoring [6, 9, 69].

In these settings, analysts need interactive and accurate estimates of diverse summary statistics across multiple data subpopulations of their data. For instance, video analysts want to monitor different statistics of viewer quality across subpopulations of viewers (e.g., entropy of bitrate in each major US city, etc) [60]. Similarly, network operators want to analyze traffic grouped by combinations of their 5-tuple (srcIP, dstIP, srcPort, dstPort, protocol) [69]. This is analogous to classical OLAP cube applications where the number of cube vertices grows exponentially as more subpopulations are admitted.

In such multidimensional telemetry settings, we ideally want frameworks offering high fidelity and interactive estimates at low operational cost. However, there are two fundamental challenges. First, there is a combinatorial explosion of data subpopulations to monitor, which can result in exponential overhead in operational costs and resources. Second, estimating multiple statistics entails compute and/or memory overhead proportional to the number of statistics of interest.

We find that existing frameworks are fundamentally limited in terms of the tradeoff across operational cost, accuracy, and estimation latencies they can offer. Exact analytics frameworks (e.g., Spark [96], Hive [84], Druid [93]) that rely on horizontal resource scaling entail poor cost-performance tradeoffs as datasets become larger. While approximate analytics [36] (e.g., sampling- or sketch-based analytics) can trade off estimation accuracy for lower cost and improved interactivity, these too suffer undesirable tradeoffs. For instance, sampling-based approaches provide generality across metrics and can handle many subpopulations, but their accuracy guarantees can be weak. On the other hand, sketch-based analytics (e.g., [18, 19, 37, 46, 47, 85, 86, 95]) can offer robust accuracy guarantees, but cannot address the combinatorial explosion of data subpopulations and also incur per-statistic effort.

In this paper, we present Hydra, a framework for efficient and general analytics over multidimensional data streams. Hydra builds on
the novel combination of two key ideas. First, to tackle the combi-
natorial explosion of subpopulations, we use a “sketch of sketches”
that enables memory efficient data stream summarization. This
reduces the framework’s data-resident memory footprint by one
to two orders of magnitude compared to Spark- and Druid-based
alternatives and offers robust and provable accuracy guarantees.
Second, to provide high-fidelity estimations simultaneously for
many statistics, we leverage universal sketching [69]. Unlike canoni-
cal sketch-based approaches that deploy one custom sketch type
per statistic [47, 85, 86], a universal sketch estimates multiple dif-
cent summary statistics with only one sketching instance.

To the best of our knowledge, Hydra is the first work to: (a) propose
the combination of a sketch-of-sketches with universal sketching
for the multidimensional telemetry problem. While some prior
works have proposed the concept of a sketch-of-sketches, they do
so for more narrow estimates of interest and do not demonstrate
practical system implementations supporting a broad range of esti-
mates (e.g., [38]); (b) analytically prove the theoretical guarantees
of such a construction; and (c) design a practical end-to-end sys-
tem design and implementation of this idea using the theoretical
analysis. We build a prototype Hydra on Apache Spark but note
that our core design is platform agnostic and can be ported to other
streaming/batching systems as well [5, 82, 93]. We also implement
practical optimizations to mitigate compute bottlenecks to further
reduce Hydra’s runtime and cost.

We evaluate Hydra using two real-world datasets: (1) a 2h-long,
January 2019 CAIDA trace from the equinix-NYC vantage point [7,
10] and (2) an anonymized real-world trace of video QoE from a
video analytics provider capturing the perceived QoE of viewers of
a US-based content provider [12]. To further evaluate the sensitivity
of Hydra-sketch, we also leverage a synthetic multidimensional
data set drawn from a Zipf distribution with different parameter
values [46, 86].

We compare Hydra against six baselines: A native Spark-SQL im-
plementation for exact analytics, a Spark-based implementation
that uniformly samples incoming data, a sketch-based approach
that allocates one universal sketch instance per-subpopulation, Ver-
dictDB [78] (a sampling-based alternative) and two key-value based
implementations (on Apache Spark and Druid) that pre-aggregate
data at ingestion time and provide precisely accurate analytics.

Our evaluation shows that: (1) Hydra offers robust accuracy (mean
error across statistics ≤ 5% with 90% probability) at 1/10 of the opera-
tional cost of exact analytics frameworks; (2) Hydra’s configuration
heuristics ensure close to optimal accuracy-memory tradeoffs; (3)
Hydra’s memory footprint scales sub-linearly with dataset size
and number of data subpopulations. Combined with performance
optimizations that improve end-to-end runtime by 45%, Hydra
offers 7-20× better query latency than Spark- and Druid-based
alternatives.

2 BACKGROUND AND MOTIVATION

In this section, we present several motivating scenarios, introduce
key aspects of multidimensional telemetry, and discuss the limita-
tions of existing analytics frameworks.

2.1 Motivating Scenarios

Video Experience Monitoring: To maintain their ad- and/or
subscription-driven revenues, video providers need to detect is-
issues that can degrade viewer experience. To that end, analysts first
collect video session summaries (i.e., per viewer measurements
of video quality) and use them to periodically (e.g., every minute)
compute summary statistics of various video quality metrics. This
allows them to monitor viewer experience across multiple subpop-
ulations of viewers [3, 60, 61]. For instance, to track the entropy
of bitrate and the L1 Norm of buffering ratio – a common indicator
of streaming anomalies – for viewers in different cities, analysts may
want to estimate the following query:

SELECT City, Entropy(Bitrate), L1Norm(Buffering)
FROM SessionSummaries
GROUP BY City

Network Flow Monitoring: Network operators commonly rely
on control-plane telemetry frameworks [41, 51] for tasks such as
traffic engineering [43, 69], attack and anomaly detection [81] or
forensics [92]. These frameworks periodically monitor performance
metrics (e.g., flow distributions, per-flow packet sizes, latency, etc.)
across different subpopulations of flows, i.e., network flows grouped
across combinations of packet header fields. For instance, the oper-
ator might want to track indicators of DDoS attacks as follows:

SELECT dstIP, Cardinality(srcIP)
FROM FlowTrace
GROUP BY dstIP

These use cases share a problem structure that is characteristic of
multidimensional telemetry. Queries that involve estimating many
statistics across many data subpopulations appear in various set-
tings, such as A/B testing [56, 62], exploratory data analysis [26, 87],
operations monitoring [16], and sensor deployments [94].

2.2 Requirements and Goals

Drawing on these use cases, we derive three key properties of the
telemetry problem we want to tackle:

1. Multidimensional Data: We define a multidimensional data
record as \( x = (d_1, \ldots, d_m, m) \), where \( d_i \) is the value of a
dimension \( D_i \) and \( m \) is the value of metric \( M \). In video, quality
metrics might be bitrate or buffering time whereas dimensions
might be the viewer’s location, their player device, their ISP
or CDN. Metrics and dimensions are domain- and use-case
specific.

2. Analytics on Data Subpopulations: Analytics are estimated
in parallel across subpopulations of the input data. A sub-
population \( Q \) is a collection of data records \( \{x_i\} \) such that all
\( x_i \in Q \) match on a subset of dimension values. With a
slight abuse of notation, we define \( Q \) using this set of di-
mension values, i.e., \( Q = \{D_1 = d_1 \land \cdots \land D_m = d_m\} \),
where \( \{D_1, \ldots, D_m\} \subseteq \{D_1, \ldots, D_m\} \); e.g., a data subpop-
ulation could be NYC-based viewers using AppleTV.

3. Multiple statistics to estimate: For each subpopulation, the
operator wants to estimate various summary statistics such as,
heavy hitters, entropy, cardinality, etc. A query \( q_k \) specifies a
set of subpopulations \( \{ Q_i \} \) and a statistic \( g \) to estimate using the values \( m_j \) of \( x_j \in Q_i \).

In practice, operators have three requirements: (1) High fidelity for a broad set of statistics i.e., robust, apriori configured, error bounds for as many statistics as possible; (2) Near real-time estimations and; (3) Low footprint (e.g., cloud compute and memory costs).

### 2.3 Prior Work and Limitations

Prior work has focused on developing two broad (but not mutually exclusive) approaches for multidimensional telemetry. The first enables distributed computations by horizontally scaling the framework’s resources. The second enables approximate analytics that sacrifice estimation accuracy for improved performance.

1. **Horizontal resource scaling**: Here we find SQL and NoSQL analytics frameworks whose distributed design reduces estimation latency through horizontal scaling of server resources (e.g., Spark [96], Hive [84], Hadoop [82], Dremel [71], Druid [93], Flink [32]). These frameworks scale their clusters with input data and can provide precisely exact estimations. However, as data volume and dimensionality grow, i) deploying such clusters becomes increasingly expensive and ii) the continuous addition of resources eventually results in marginal estimation latency gains due to data shuffling overheads [20].

2. **Approximate Analytics**: Approximate analytics frameworks leverage sampling or data summarization algorithms in order to trade off accuracy for performance and cost. Sampling-based frameworks allow for low estimation latency by sampling data either online, at query time (the analyst applies the sampling operators and parameters as part of the estimation query) [24, 33, 71, 76, 84] or offline, by means of a pre-processing step that creates data samples to be used at query time [17, 18, 20, 83]. While there is a rich body of sampling-based efforts, these have two key shortcomings. First, their accuracy guarantees are in the form of confidence bounds that are computed after query estimation has taken place and that depend on the statistic being estimated and the number of samples used [70]. Therefore, when an estimate does not meet accuracy requirements, frameworks often fall back to using other samplers or precisely exact estimates. Second, to offset the resource overheads of producing offline samples, frameworks often make hard apriori choices on what subsets of their data to create samples for (e.g., BlinkDB [20] that mines query logs for frequently queried data or VerdictDB [78] that allows users to identify popular data tables). In contrast, sketch-based analytics ensure bounded accuracy-memory trade-offs for arbitrary workloads in sub-linear space [39, 40, 42, 69, 95]. These frameworks build compact data summaries at ingestion and use them to estimate statistics with apriori provable error bounds.

We can also combine horizontal resource scaling and approximations. For instance, both Apache Spark and Druid allow for data summarization at ingestion time such that incoming data are stored as a key-value store where the keys are distinct \( (Q_i, m_j) \) tuples and the values are their respective counts. These hybrid approaches enable data reduction without compromising the framework’s ability to offer precise estimations.

#### Qualitative Analysis:

Next, we analyze the overhead to process multidimensional streams and the resident data cost using the above solutions. Let us denote the dataset size (in terms of the number of data records) as \( V \) and let \( Q \) be the number of data subpopulations. Assuming \( D \) dimensions and that each data record belongs in \( 2^D \) different subpopulations, then \( Q = O(2^D \times V) \). In practice, assuming each dimension has cardinality \( C \), there are \( O(C^D) \) subpopulations in the dataset. In practice, we find that \( O(2^D \times V) \) is a tighter empirical bound for \( Q \) and we will use that moving forward. In addition, given that the framework needs to estimate \( O(S) \) different statistics, the number of summary statistics to be estimated is an exponential \( O(Q \times S) = O(2^D \times V \times S) \). Assuming, as it is the case for frameworks for precisely exact analytics, that the CPU and memory requirements for data ingestion and/or statistics estimation scale linearly with subpopulations, we see that the framework’s runtime, resource requirements and cost also scale exponentially.

#### Quantitative Analysis:

To corroborate this qualitative analysis, we evaluate the operational cost for several analytics frameworks when used in a multidimensional context (Figure 1). Specifically, we measure their cost as a function of their observed accuracy when asked to estimate in real-time 4 summary statistics from a 130GB real-world dataset with approximately 5.6 million data subpopulations. Following the typical cloud billing model [11], we use the total runtime \( \times \) the number of cluster nodes used (20) as a proxy for the cost. We provide a detailed description of our experimental setup and baselines in §6. Ideally, we need a framework whose cost-accuracy tradeoff lies in the top-left, green region, i.e., it offers the accuracy of a precise analytics framework at the cost of sampling. However, we observe that the cost gap between the cheapest (1% uniform sampling) and the most expensive baselines (precisely accurate Spark-SQL) is two orders of magnitude wide. A sketch-based approach where the framework allocates one sketch per subpopulation, while cheaper than Spark-SQL, remains expensive as it allocates exponentially many sketch instances, thus incurring high memory overheads. As discussed in §6, this baseline uses universal sketching that can simultaneously estimate all 4 statistics of interest per subpopulation with one sketch. Finally,
precise baselines that summarize data at ingestion time, such as Apache Druid and Spark (denoted as Spark-KV) lie in the middle between Spark-SQL and sampling.

**Key takeaways:** Multidimensional telemetry entails a combinatorial explosion of data subpopulations and summary statistics to monitor. Balancing cost, accuracy, and estimation latency is challenging due to the combinatorial explosion in data subpopulations and the number of summary statistics the framework needs to enable. Existing frameworks can only meet a subset of these goals, which motivates us to rethink how to support such analytics workloads at scale.

3 **HYDRA: SYSTEM OVERVIEW**

To support multidimensional workloads at scale, we envision HYDRA as a streaming, sketch-based OLAP framework [4, 93]. HYDRA’s distributed design (illustrated in Figure 2) includes one frontend and multiple worker nodes and its input are i) streams of multidimensional data, ingested in parallel at the worker nodes and ii) estimation queries provided by the operator to the frontend node. HYDRA implements two logical operations: Data Ingestion and Query Estimation.

(1) **Data Ingestion:** Data Ingestion happens at the worker nodes. Each worker summarizes an incoming data stream to a local instance of HYDRA-sketch. Data summarization happens on a per-subpopulation basis. Specifically, for every incoming data record, HYDRA first identifies what subpopulations the data record belongs in and correspondingly updates a novel sketching primitive that we discuss below, HYDRA-sketch. HYDRA-sketch instances are configured to ensure accuracy guarantees and low memory footprint (§4.6).

(2) **Query Estimation:** Query Estimation involves both frontend and worker nodes. The frontend receives operators’ queries with i) the statistics to estimate and ii) the set of subpopulations to estimate these statistics on. Using this information, it creates a query plan that is distributed to the worker nodes who execute the queries. After estimation has taken place, the frontend node collects the results from the workers and returns them to the operator.

While the idea of using sketching to optimize analytics is not new, in our context canonical sketch-based approaches will need to instantiate up to $O(S)$ sketch instances per subpopulation. This is inefficient as the framework needs exponentially many sketch instances, despite a sketch’s ability to summarize a subpopulation’s data in sub-linear space.

**Key Idea:** To avoid the above limitations of conventional approaches, HYDRA uses a novel combination of two ideas.

First, we observe that we can reduce the exponential $O(Q) = O(2^D \times V)$ ingestion-time, memory cost of sketch-based approaches through a novel “sketch of sketches”. We show that through a $w \times r$ array of sketch instances (Fig. 3), where $w \times r \ll 2^D \times V$, HYDRA reduces the memory cost of estimating $O(S)$ statistics from $O(2^D \times V \times S)$ to $O(wr \times S)$. The intuition is that, unlike canonical sketch-based approaches, we can summarize multiple subpopulations into one sketch instance and then query it with predictable error [38, 86].

Second, to reduce the need for instantiating $O(S)$ different sketch types for $O(S)$ summary statistics, HYDRA leverages universal sketching [30, 69]. Universal sketching enables replacing $O(S)$ sketches with a single sketch that simultaneously estimates multiple different statistics per subpopulation. This means that as long the desired statistics can be estimated with a universal sketch, there is no limit in the number of statistics that the sketch can estimate with a fixed memory footprint. This design choice further reduces the framework’s space complexity from $O(wr \times S)$ to $O(w \times r)$.

Figure 2: HYDRA’s example workflow. Workers perform data ingestion and querying. The frontend node exposes the query API to the operator and performs configuration and query plan dissemination.

Figure 3: Comparison of Ingestion and Estimation (CPU time, space complexity) for different sketch-based designs. We highlight the theoretical improvements in space complexity from HYDRA’s design ideas.

While these two ideas (sketch of sketches and universal sketching) have been independently proposed in other narrower contexts, to the best of our knowledge, we are the first effort to: (1) propose the combination of these ideas to tackle the multidimensional telemetry problem; (2) rigorously prove the accuracy-resource tradeoffs of this construction; and (3) demonstrate a practical end-to-end realization atop state-of-art horizontally scalable "BigData" platforms.
Table 1: HYDRA Notation. The upper subsection introduces notation specific to the sketch-of-sketches and the lower to universal sketches.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$V$</td>
<td>Input size</td>
</tr>
<tr>
<td>$D$</td>
<td>Number of data dimensions</td>
</tr>
<tr>
<td>$Q$</td>
<td>Number of data subpopulations</td>
</tr>
<tr>
<td>$S$</td>
<td>Number of summary statistics</td>
</tr>
<tr>
<td>$S_{m,n}$</td>
<td>Stream of length $m$ and $n$ distinct keys</td>
</tr>
</tbody>
</table>

| $w$      | Number of sketches per 2D-sketch row |
| $r$      | Number of rows in 2D-sketch |
| $(\epsilon, \delta)$ | $0 < \epsilon < 1$ as additive error and $\delta$ is the probability that the result error is not bounded by $\epsilon$ (failure probability) |

| $w_{US}$ | Number of counters per universal sketch row |
| $r_{US}$ | Number of universal sketch rows |
| $(\epsilon_{US}, \delta_{US})$ | $0 < \epsilon_{US} < 1$ as additive error in universal sketch and $\delta_{US}$ is the failure probability |
| $L$      | Number of universal sketch layers |
| $k$      | Number of keys in universal sketch heavy hitter heaps |

4 HYDRA DETAILED DESIGN

We first provide background on sketching to set up the intuition for HYDRA-sket. We then introduce the basic HYDRA-sketch algorithm, formally prove its error bounds, and devise HYDRA-sketch configuration strategies. Table 1 summarizes the notation we use.

4.1 Background on Sketching

Let $S_{m,n}$ denote a data stream with length $m$ and $n$ distinct keys. Suppose we want to estimate a frequency-based summary statistic of the keys (e.g., entropy, cardinality, frequency moments). A natural design is to estimate the desired statistic with a key-value data structure tracking the frequency per key. For instance, for frequency estimation, we can maintain and increment one counter per key. While correct, the space complexity is linear in $n$ and not space efficient (Figure 4).

**Figure 4: Maintaining per-key state is not space efficient**

Hash-based mappings for space efficiency: To ensure sub-linear (in $n$) space complexity, sketching algorithms do not maintain per-key state but, instead, map multiple keys to the same counters via hashing. For instance, a simple sketch for frequency estimation consists of $w$ integer counters, where $w \ll n$. Based on the hash of the key, an element gets mapped to a counter, which is then incremented to maintain an estimate of that key’s frequency. Naturally, multiple keys colliding introduces some errors (Figure 5).

Multiple independent updates for tighter error bounds: As defined, this basic mechanism only provides a small probability that the estimation error will lie within a desirable range of error values [22]. To overcome this, sketches use independent instances (e.g., $r$ arrays) of the counter structure of length $w$. Each vector of length $w$ has its own hash function and the $w$ hash functions are pairwise independent. Thus, ingesting a stream element now translates to $r$ update operations (e.g., incrementing $r$ integer counters instead of one). For each key, this sketch produces $r$ different estimates of the statistic of interest. The final estimate will be a sum of $r$ estimates (i.e., min, median etc.) (Figure 6) [39]. This amplifies the probability that the estimation error lies within the desired range.

**Figure 5: Hashing enables sub-linear memory complexity**

**Figure 6: Independent hashing improves accuracy.**

4.2 Tackling Subpopulation Explosion

For now, let us make the simplifying assumption (which we relax later) that our system only needs to estimate one summary statistic (e.g., entropy) per data subpopulation. Similar to Figure 4, a starting point for our design would be to maintain per-subpopulation state, i.e., allocate one sketch instance for each of the $O(2^D \times V)$ distinct subpopulations. This approach, similar to an OLAP cube, is not scalable as it requires as many sketches as the number of data subpopulations.

To avoid keeping per-subpopulation state, we borrow from the first intuition that we saw in the sketch construction in the background (Figure 5). The basic sketch construction avoids maintaining per-key state by allowing multiple keys to explicitly collide in a hashed key-value store whose size is less than the number of unique elements.

Note that the basic sketch is maintaining a single counter per array entry but we want to be able to estimate some statistical summary of a subpopulation instead. Therefore, instead of keeping a single counter per array entry, we maintain a sketch-per-entry. This brings us to the following construction (Figure 7). We consider a single array of $w$ (e.g., $w \ll 2^D \times V$) sketches. For each ($Q_i$, $m_j$) pair, we hash the $Q_i$ and map it to one of the $w$ sketches, thus colliding multiple subpopulations to the same sketch. Then, we update the sketch with $m_j$ and at query time, we estimate the statistic for $Q_i$.

Analogous to the basic sketch from §4.1, by mapping multiple subpopulations to one sketch, this baseline construction will have some
functions, known as Stream-PolyLog [29, 30, 69]. We denote each function in Stream-PolyLog, as $G$-sum, where $f_j$ is the frequency of the $j$-th unique element in the input stream $S_{m,n}$ and $g$ is a function defined over $f_j$. If $g$ is monotonically increasing and upper bounded by $O(f_j^2)$, then $G$-sum can be computed by a single universal sketch with polylogarithmic memory. Universal sketch provides $\epsilon$-additive error guarantees to Stream-PolyLog and demonstrates better memory-accuracy tradeoffs than the composition of custom sketches when estimating multiple statistics from Stream-PolyLog in practice [69]. Key statistics of interest can be formulated via a suitable $G$-sum in Stream-PolyLog. Such examples include: $\alpha$-Heavy Hitters ($f_j \geq \alpha f_j$), L1-Norm ($\sum |f_j|$), L2-Norm ($\sum f_j^2$) Entropy ($-\sum \frac{f_j}{L} \log \frac{f_j}{L}$), and Cardinality ($\{|f_1, \ldots, f_N|\}$). Note that there are many other summary statistics that can be estimated by combining statistics, such as standard deviation, histograms, mean, or median. A statistic that cannot be directly estimated by HYDRA-sketch is quantiles.

The basic building block of universal sketches are L2-Heavy Hitter (HH) sketches e.g., Count-sketch [72]. Each count-sketch maintains $r_{CS}$ arrays of $w_{CS}$ counters each, $r_{CS}$ pairwise-independent hash functions and a max-heap keeping track of the top-$k$ Heavy Hitters in the sketch; When updating each count-sketch with a new data item, the sketch updates a randomly located counter in every row based on the corresponding hash index to keep track of that data item’s frequency. The top-$k$ HH heap is subsequently updated to reflect the addition of the new item. A universal sketch consists of $L$ layers of count-sketches. Each count sketch applies an independent 0-1 hash function $h_{CS}(i, L_1)$ to the input data stream to sub-sample at every layer (from the previous layer). These layers then track the heavy hitters, i.e., the important contributors to the $G$-sum.

The intuition here is that the layered structure of the universal sketch is designed for sampling representative elements with diverse frequencies, and these elements can be used to estimate $G$-sum with bounded errors. If only one layer of heavy hitter sketch were used, the estimations would lack representatives from less frequent elements. The heavy-hitters at each layer are processed iteratively from the bottom layer to the top and the recursively aggregated result is used to compute the desired statistic. This is an unbiased estimator of $G$-sum with bounded additive errors (Theorem 1).

**Theorem 1** ([30, 69]). Given a stream $S_{m,n}$, let us consider a Universal Sketch US with $L = O(\log n)$ layers. If each layer of US provides an $(\epsilon_{US}, \delta_{US})$-L2 error guarantee, then US can estimate any $G$-sum function $G \in \text{Stream-Polylog}$ to within a $(1 \pm \epsilon_{US})$ factor with probability $1 - \delta_{US}$. Satisfying an $(\epsilon_{US}, \delta_{US})$-L2 error guarantee requires $O(\log n)$ Count Sketch instances with $w_{CS} = O(\epsilon_{US}^2)$ columns and $r_{CS} = O(\log \frac{1}{\delta_{US}})$ rows.

### 4.4 The HYDRA-sketch Algorithm

Combining these ideas gives us the HYDRA-sketch algorithm.

1. **Updating HYDRA-sketch:** Updating HYDRA-sketch with a data record, $x_i = \langle d_{i,1}, d_{i,2}, \ldots, d_{i,m} \rangle$ is a three-step process. At the first, “fan-out” stage, we compute the $O(2^D)$ subpopulations
The quantity which we wish to estimate is \(G\), groups as \(\mathbb{Q}\) the \(\mathbb{G}\) mapped to. Let \(\mathbb{Q}\) hash function \(h_\mathbb{Q}\). For the \(k\)th row, the index of the universal sketch to update \(\mathbb{US}_k\) is the hash of \(\mathbb{Q}\) using hash function \(h_\mathbb{Q}\). Last, we update each \(\mathbb{US}_k\) with the metric value \(m_j\).

(2) **Querying Hydra-sketch:** Hydra-sketch’s querying algorithm takes as input a statistic \(g\) and an aggregation \(\mathbb{Q}\); i.e., the aggregation to estimate \(g\) on. Querying consists of 2 steps. The first involves identifying the set of \(\mathbb{US}_k\) that \(\mathbb{Q}\) maps to. Then \(g\) is estimated from each \(\mathbb{US}_k\) and the median value of these estimations is returned.

Given this basic algorithm, we now focus on formally proving that Hydra-sketch offers rigorous accuracy guarantees and that it is usable in practice.

### 4.5 Accuracy Guarantees

**Theorem 2.** Let us assume that each Universal Sketch \(\mathbb{US}\) can approximate the \(G\)-sum, for a monotone function \(g\) within a \((1 + \epsilon_{US})\)-factor with probability \(1 - \delta_{US} > 1/2\). Further, let \(\mathbb{G}_0\) be the \(G\)-sum applied to the stream \(\mathbb{S}\) and \(G_i\) when applied to the target subpopulation \(\mathbb{Q}\). Then Hydra-sketch with \(w = O(e^{-1})\) columns and \(r = O(\log \delta^{-1})\) rows, for user defined parameters \(\epsilon, \delta\), provides an estimate \(\hat{G}_i\) that with probability \(1 - \delta\) satisfies:

\[
G_i(1 - \epsilon_{US}) \leq \hat{G}_i \leq G_i(1 + \epsilon_{US}) + \epsilon \cdot G_0
\]

**Proof.** To bound the error of our algorithm, we analyze the frequency vector \(f_j\) of the stream of elements mapped to each Universal Sketch instance \(US_j = h_j(\mathbb{Q})\), where \(\mathbb{Q}\) is the queried subpopulation. The frequencies of all \(m_j \in \mathbb{Q}\) are guaranteed to appear in \(f_j\), since the Update Algorithm of §4.4 maps them to \(US_j\).

Let \(\mathbb{Q} = \{Q_1, \ldots\}\) denote all groups in the input stream \(\mathbb{S}\), and let \(\mathbb{Q}_j = \{Q_k \in \mathbb{Q} | h_j(Q_k) = h_j(\mathbb{Q}_j)\}\) denote the set of groups mapped to \(US_j\). That is, \(G_j = \sum_{Q_k \in \mathbb{Q}_j} \sum_{m_i \in Q_k} g(f_{m_i})\).

The quantity which we wish to estimate is \(G_j = \sum_{x \in Q_k} g(f_{m_i})\), i.e., the \(g\)-sum of the group \(Q_k\), while the \(US_j\) processes all groups in \(\mathbb{Q}_j\) and thus approximates \(\sum_{Q_k \in \mathbb{Q}_j} \sum_{m_i \in Q_k} g(f_{m_i}) = G_j + \sum_{Q_k \notin \mathbb{Q}_j} \sum_{m_i \in Q_k} g(f_{m_i})\). For all \(j \in \{0, \ldots, r - 1\}\), denote by \(\hat{G}_{i,j}\) the estimate of \(US_j\), and denote the noise added by the other groups as \(N_j = \sum_{Q_k \notin \mathbb{Q}_j} \sum_{m_i \in Q_k} g(f_{m_i})\). Notice that, since any group in \(\mathbb{Q} \setminus \{Q_i\}\) has a probability of \(1/w\) of being in \(\mathbb{Q}_j\), its expectation satisfies that: \(\mathbb{E}[N_j] = \sum_{Q_k \notin \mathbb{Q}_j} \sum_{m_i \in Q_k} g(f_{m_i}) \leq \frac{G_0}{w}\). Therefore, according to Markov’s inequality, for any \(c \in \mathbb{R}^+\), \(\Pr[N_j \geq c \cdot \frac{G_0}{w}] \leq 1/c\). Next, by the correctness of the universal sketch, we have that,

\[
\Pr[\hat{G}_{i,j} \notin [(G_i + N_j)(1 - \epsilon_{US}), (G_i + N_j)(1 + \epsilon_{US})]] \leq \delta_{US}.
\]

Since \(g\) is part of \(G\)-sum in \(Stream\)-\textit{PolyLog}, it must be monotone, and thus \(N_j \geq 0\). This means that with probability of at least \(1 - \delta_{US} - 1/c\) both \(\hat{G}_{i,j} \in \{G_i(1 - \epsilon_{US}), (G_i + N_j)(1 + \epsilon_{US})\}\) and \(N_j < c \cdot G_0/w\) simultaneously hold, and thus

\[
G_i(1 - \epsilon_{US}) \leq \hat{\hat{G}}_{i,j} \leq G_i(1 + \epsilon_{US}) + \frac{c}{w} + \epsilon \cdot G_0.
\]

Therefore, we pick \(w = c \cdot (1 + \epsilon_{US}) \cdot e^{-1}\) and a \(\epsilon\) value such that \(1 - \delta_{US} - 1/c > 1/2\), to get that

\[
\Pr[G_i(1 - \epsilon_{US}) \leq \hat{\hat{G}}_{i,j} \leq G_i(1 + \epsilon_{US}) + \epsilon \cdot G_0] > 1/2
\]

Recall that the algorithm’s query sets \(\hat{G}_i = \text{median} \hat{G}_{i,j}\) and that the \(r\) rows are i.i.d. and thus a Chernoff bound yields

\[
\Pr[G_i(1 - \epsilon_{US}) \leq \hat{\hat{G}}_i \leq G_i(1 + \epsilon_{US}) + \epsilon \cdot G_0] \geq 1 - \delta.\]

**Takeaways:** We note the following from Theorem 2. The error bounds of Hydra-sketch are tunable based on the choice of its configuration parameters that control \((\epsilon, \delta)\) and \((\epsilon_{US}, \delta_{US})\). In addition, the upper error bound is additive, which means that it will allow for loose error bounds in cases where \(\epsilon \cdot G_0 \approx G_i\). We discuss these takeaways in more detail below.

### 4.6 Hydra-sketch Configuration

![Figure 9: Hydra-sketch structure and configuration parameters.](image)

We now focus on techniques to tune Hydra-sketch’s parameters. As illustrated in Figure 9, Hydra-sketch has six configuration parameters: two parameters \((r, w)\) define the structure of the sketch arrays and additional four \((L, W_{CS}, r_{CS}, k)\) determine the inner structure of the Universal Sketches. The choice of configuration parameters of Hydra-sketch affects its empirical accuracy and memory footprint. For instance, larger \(w\) and \(r\) values ensure better estimation accuracy but require more memory.

It is often useful to reason about the relative error of the estimation; rephrasing Theorem 2, we can write:

\[
\Pr[\frac{G_i - \hat{G}_i}{G_i} \leq \epsilon_{US} + \epsilon \cdot \frac{G_0}{G_i}] \geq 1 - \delta.
\]

and thus

\[
\Pr\left[\frac{\hat{\hat{G}}_i - G_i}{G_i} \leq \epsilon_{US} + \epsilon \cdot \frac{G_0}{G_i}\right] \geq 1 - \delta.
\]

That is, we have that with probability \(1 - \delta\), the relative error is at most \(\epsilon_{US} + \epsilon \cdot \frac{G_0}{G_i}\). Since \(\epsilon_{US}, \epsilon, \) and \(G_0\) are determined by the configuration and not a specific subpopulation, we get that the relative error bound is looser if \(G_i\) is small. Intuitively, if a
subpopulation is very small, the noise we get from the colliding subpopulations may be larger than its own statistics.

With that in mind, we consider a quantity \( G_{\text{min}} \) that denotes the minimal \( G \)-sum for which we want to guarantee some relative error, e.g., of 20%, with a high probability, e.g., 90%. This means that for any subpopulation with a higher \( G \)-sum, the error is upper bounded by \( \epsilon_{US} + \epsilon \cdot \frac{G}{G_{\text{min}}} \). This allows us to derive configuration heuristics for Hydra-sketch as follows:

Controlling the probability of error bounds holding: From Theorem 2, for the error bound of our example to hold with 90% probability, \( 1 - \delta = 0.9 \) and, hence, \( \delta = 0.1 \). This translates to \( r = 3 \).

Similarly, from Theorem 1, a universal sketch will estimate any \( G \)-sum function within an \( \epsilon_{US} \) factor with probability \( 1 - \delta_{US} \). For probability 90%, \( \delta_{US} = 0.1 \) and, thus, \( r_{US} \approx 3 \).

Minimizing upper error bound: To minimize the upper error bound of Hydra-sketch, we need to minimize \( E = \epsilon_{US} + \epsilon \cdot \frac{G}{G_{\text{min}}} \) under a memory constraint, \( O(M) = w \times w_{US} \). From Theorems 1 and 2, we know that \( \epsilon \approx 1/w \) and \( \epsilon_{US} \approx 1/\sqrt{w_{US}} \). This allows us to minimize \( E \) for \( w \) and \( w_{US} \) as follows:

1. Solving for \( \epsilon_{US} \): Given the memory constraint, we can write \( E = \epsilon_{US} + \epsilon \cdot \frac{G_{\text{US}}}{G_{\text{min}}} \). Minimizing \( E \) over \( \epsilon_{US} \) gives us:
   \[
   \epsilon_{US} = \sqrt{\frac{2G_{\text{US}}}{MG_{\text{min}}} \Rightarrow w_{US} = \Theta\left(\frac{MG_{\text{min}}}{G_{\text{US}}}\right)^{2/3}.}
   \] (3)

2. Solving for \( \epsilon \): Similarly, we can write \( E = \sqrt{\frac{G_{\text{US}}}{M}} + \epsilon \cdot \frac{G}{G_{\text{min}}} \). Minimizing over \( \epsilon \) gives:
   \[
   \epsilon = \left(\frac{2\sqrt{MG_{\text{US}}}}{G_{\text{min}}} \right)^{-2/3} \Rightarrow w = \Theta\left(\frac{\sqrt{M}G_{\text{US}}}{G_{\text{min}}} \right)^{2/3}
   \] (4)

Controlling remaining universal sketch parameters: Last, we configure the levels \( L \) maintained in each universal sketch instance and the number of heavy keys \( k \) needed to store at each level’s heavy hitter heap. From Theorem 1, \( L = \Omega(\log n_{US}) \), where \( n_{US} \) is the average number of distinct subpopulations summarized at a universal sketch. For the value of \( k \), we empirically set its lower bound to \( k = \Omega(1/\epsilon_{US}^2) \). For \( \epsilon_{US} = 0.1 \), this translates to \( k \approx 100 \).

Let us now see how we can use these guidelines in practice. As an example, let us assume we want the relative error of estimation to not exceed 0.2 with 90% probability for subpopulations where \( G_i/G_2 \geq 10^{-3} \). Thus, \( G_{\text{min}} \geq 10^{-3} \cdot G_2 \). Let us also assume that \( \epsilon_{US} = \epsilon \cdot \frac{G_{\text{US}}}{G_{\text{min}}} = 0.1 \). From Eq. (3), we can get an estimate of memory needed, \( M \approx 10^6 \) needed. Note that here \( M \) measures “units of \( w_{US} \)”, i.e., counters. Thus, \( \epsilon_{US} = 0.1 \) and \( w_{US} = \Theta(10^2) \). From \( O(M) = w \cdot w_{US} \), we can further see that also \( w = \Theta(10^2) \).

In §6, we show that these strategies can achieve near optimal trade-offs. We acknowledge that implementing this workflow assumes that the operator has some prior knowledge about the workload i.e., a rough estimate of the number of subpopulations. We believe this is not an unreasonable requirement in many practical settings.

5 IMPLEMENTATION

This section discusses our implementation of Hydra and the practical performance challenges we faced. Our prototype of Hydra-sketch can be found in [13].

Baseline Implementation and Workflow: We implement Hydra’s workflow (§3) on top of Apache Spark [96] as Spark’s extensibility allowed us to easily prototype design alternatives. However, Hydra’s workflow can easily fit into different analytics frameworks e.g., Druid [93].

Data ingestion happens at the worker nodes. Each worker node splits its input into ~64MB partitions, allocates one Hydra-sketch instance per partition and updates it with that partition’s data. We implement these and Hydra-sketch instances as Spark RDDs. To allocate appropriately configured Hydra-sketch instances, workers rely on configuration manifests distributed by the frontend node.

As a result of splitting input data into smaller batches, each worker node maintains multiple instances of Hydra-sketch. The design of Hydra enables sketch merging due to the well-known linearity property of frequency-based sketches. Therefore, during data ingestion, worker nodes merge Hydra-sketch instances of fully ingested partitions until Hydra is left with one Hydra-sketch instance to query. For sketch merging, we use Spark’s “treeAggregation” module [1], thus mitigating the risk of performance bottlenecks.

Query estimation involves both the frontend and the worker nodes. The operator inputs the desired queries and the frontend then generates a query plan for the worker nodes to execute. Estimation results are collected at the frontend node.

An accuracy-improving heuristic: Recall from §4.4 that after \( Q_i \) is mapped to a universal sketch, that sketch only stores the frequencies of metric values \( m_j \). This design, however, does not keep track of which subpopulation \( Q_i \) each \( m_j \) maps to. As a result, a universal sketch will return the same estimations for all subpopulations whose data it stores. Our heuristic is simple: Instead of updating each universal sketch with \( m_j \), we can use a more fine-grained key, i.e., the concatenation of the metric value and its corresponding subpopulation. This way, heavy hitter heaps will maintain heavy counts for each \( (Q_i, m_j) \) pair and will be able to differentiate between them.

Implementation optimizations: To further reduce the system’s runtime, we introduce a few optimizations:

1. **One Large Hash per \( (Q_i, m_j) \) Pair:** Updating Hydra-sketch \( (Q_i, m_j) \) requires \( O(r \times L) \) hash computations, \( r \) to identify the universal sketches to update and up to \( L \) per universal sketch. We reduce the number of hashes to \( O(1) \) by computing one large 128-bit hash and breaking it down into substrings of variable lengths and treating each substring as a separate hash. Prior analysis [44, 64] shows that different substrings from the same long hash provide sufficient independence.

2. **One Layer Update:** In prior universal sketching implementations, the algorithm keeps a heap to track frequent keys per layer. For each datapoint update, the universal sketching needs to update two of its layers on average. In Hydra-sketch, we follow [94] and
6 EVALUATION

We now evaluate Hydra using real-world and synthetic datasets. We provide a sensitivity analysis of our design, and evaluate our configuration strategies and optimizations. In summary:

1. Hydra offers ≤ 10sec query latencies and is 7-20× smaller than existing analytics engines.
2. Hydra offers ≤ 5% mean errors (combined across statistics) with 90% probability for a broad set of summary statistics at 1/10 of the $ cost of exact analytics engines.
3. Thanks to Hydra’s sub-linear (to the number of subpopulations) memory scaling, Hydra achieves close to an order of magnitude improvement in operational cost compared to the best exact analytics baseline.
4. Hydra’s sketch configuration strategies ensure near-optimal memory-accuracy tradeoffs.
5. Hydra’s performance optimizations improve end-to-end system runtime by 45% compared to a deployment that uses the basic Hydra-sketch design.

6.1 Experimental Methodology

Setup: We evaluate Hydra on a 20-node cluster of m5.xlarge (4CPU - 16GB memory) AWS servers [11]. In practice, we observe that nodes have ≈10-11GB of available main memory. We allocate 3 CPUs for Hydra and its input data is CSV files that are streamed from AWS S3. We configure Hydra-sketch using the heuristics of §4.6 to ensure a conservative lower error bound of -10% (i.e., ε_L = 0.1) and upper bound of 20% with 90% probability for G_{min}/G_q = 2 · 10^{-3}. We also use the performance optimizations of §5. While these bounds are conservative, they ensure a memory footprint of < 100MB per Hydra-sketch instance; our results show that the actual errors were much smaller.

Datasets: We use two real-world datasets and a synthetic trace. Each dataset maps to a different use case. First, we use CAIDA flow traces [10] collected at a backbone link of a Tier1 US-based ISP. The total trace is up to 130GB in initial size and flow data can be clustered in up to approximately 5.6M subpopulations Qi. Given that we analyze m_j metric values per subpopulation, this dataset contains up to 506M distinct (Qi, m_j) pairs. Second, we use a real-world trace of video session summaries corresponding to one major US-based streaming-video provider. The size of the video-QoE trace is approximate 5GB, with data that we cluster in up to 700k subpopulations and up to 25M (Qi, m_j) pairs. Third, we generate synthetic traces following Zipf distribution with varying skewness (e.g., 0.7 to 0.99).

Summary statistics: We evaluate Hydra’s accuracy using L1/L2 norms, entropy and cardinality i.e., statistics that map to the queries described in §2. For each subpopulation, we compute the precise value of each statistic as ground truth and then estimate the relative error with respect to Hydra’s accuracy.

Evaluation baselines: For our experiments, we compare Hydra against several baselines: From the space of precise analytics we compare with: (1) Spark-SQL: This is a traditional SQL implementation where incoming data record is stored as a row in one (logical) data table. At estimation time, we create a Key-Value store, where the keys are distinct subpopulations Qi and the values are lists of metric values m_j per subpopulation; (2) Spark-KV: Here, we summarize incoming data at ingestion time and maintain a Key-Value store where the keys are distinct (Qi, m_j) pairs and the values are their respective frequency counts; (3) Druid: This is similar to Spark-KV but uses Druid’s data roll-up feature to generate the key-value store.

From the space of approximate analytics engines, we compare against: (1) Uniform Sampling: We implement 10% uniform sampling at ingestion time and then apply the Spark-KV approach to the sub-sampled data that contains ≈82M distinct (Qi, m_j) pairs; (2) VerdictDB [78]: We deploy VerdictDB on Amazon Redshift and use the default nodes of that service (20 dc2.large nodes, each with 2CPU, 15GB memory and 160GB NVMe-SSD as storage) as backend SQL engine. VerdictDB builds offline samples, so we create hash-based sample tables for cardinality metric and uniform sample tables for L1 and L2 norm. We set sampling rate = 1% for both sample tables. VerdictDB does allow entropy estimations; (3) One Universal Sketch per subpopulation.

6.2 End-to-End Evaluation of Hydra

To evaluate Hydra end-to-end we investigate whether the system meets operators’ requirements as outlined in §2. To that end, we investigate three questions:

What is Hydra’s operating cost compared to our baselines? We measure the normalized query estimation $ cost for 4 statistics for the CAIDA dataset (130GB, 5.6M subpopulations, 506 distinct (Qi, m_j) pairs). We estimate their normalized cost as VerdictDB on Amazon Redshift constrained us to specific servers with a different pricing model.

Figure 1 depicts Hydra’s cost-accuracy tradeoff. Hydra’s cost is ~2 orders of magnitude smaller than that of Spark-SQL. That is because Spark-SQL processes the entire dataset at query time and because estimation happens at the frontend node. Hydra’s estimation cost is also an order of magnitude lower than Druid’s which uses data summaries created at ingestion. However, as we will see later, Druid’s ingestion is very inefficient. The best performing, precisely accurate baseline is Spark-KV that produces frequency counts for the resulting 506 KV-pairs at ingestion time and uses that for estimating statistics. Spark-KV is ~7× more expensive than Hydra.
Regarding approximate analytics baselines, we observe that VerdictDB, while very accurate (~98% mean accuracy for 1% sampling), exhibits large estimation times, comparable to worst-case estimation times in the original VerdictDB paper [78]. When normalized by server cost, VerdictBD’s cost is comparable to Spark-SQL. Hydra’s operational cost is on par with a sampling approach that uniformly samples 10% of all data but whose error can be very large. Perhaps surprisingly, the 10% baseline exhibits higher cost. This is because this baseline still needs to process ≈ 82M KV pairs and still requires more memory than Hydra. In the case of the smaller video-QoE dataset (not shown due to lack of space), Hydra is only 3× cheaper than Spark-SQL and approximately as costly as Spark-KV. This smaller gap is due to the smaller size of the dataset. In §6.3, we look at the empirical runtime and memory requirements that explain the observed cost results.

Is Hydra accurate and general across summary statistics? To evaluate Hydra’s accuracy and generality, we look at the accuracy of four different sets containing different numbers of summary statistics. Figure 11 depicts the boxplot of empirical estimation error for each statistic. Positive error values indicate overestimation errors and negative error values indicate underestimation. For all application sets, Hydra operates under the same resource budget and configuration as described previously. We find that estimating multiple summary statistics does not incur accuracy reduction, compared to when individual statistics are estimated. This highlights Hydra’s generality, which is enabled by the fact that information maintained in the universal sketches is statistic-agnostic and is equally used for multiple statistics of interest. Hydra’s median estimation error is almost 0 for the L2-norm, -5.7% and -5.5% for entropy and L1 norm respectively and 9.8% for cardinality estimation. We can observe that the estimated errors are well within the accuracy threshold that we set. However, for cardinality, we observe a higher median and variance in error values. This is due to a large concentration of G’s near Gmin. Recall from the discussion of §4.6 that Hydra’s error is loosest when Gi ≈ Gmin and this allows for higher error variance.

Figure 10 corroborates this observation by depicting the distribution of estimation error values for all summary statistics as a function of the subpopulation’s normalized G-sum i.e., Gi/GS. Note that for values of Gi/GS ≈ Gmin/Gs the variance of empirical error becomes larger as that is the region where the error is allowed to approach our worst-case error bound. Cardinality estimation using one universal sketch per subpopulation yields estimations with <7% error. The figure also compares Hydra with uniform sampling and highlights the high variance in error that sampling exhibits. We observe the same behavior for the video-QoE dataset with a mean error across statistics of ~6%.

6.3 Detailed Analysis of Hydra-sketch

First, we compare Hydra-sketch’s memory footprint to that of our baselines. Second, we show that our configuration strategies converge to a near-optimal configuration with respect to memory and runtime. Lastly, we show that our performance optimizations reduce Hydra’s runtime by 45%.
Memory Footprint vs. Subpopulations: Figure 13 shows memory footprint as a function of the number of subpopulations monitored for the CAIDA dataset. HYDRA follows the theoretically-expected sub-linear memory scaling as the dataset size and subpopulations increase. Indeed, while we observe that for smaller datasets, a Spark-KV implementation might be preferable in terms of memory footprint (as the size of the sketch instances might even exceed that of the input), this trend is very quickly reversed. This is an observation that is also confirmed for the video-QoE dataset.

Configuration Heuristics: Figure 14 depicts the relationship between the memory footprint of HYDRA-sketch and its estimation error for different configurations. The estimation error of the figure is that of the L1-Norm of the CAIDA dataset. The optimal configurations simultaneously minimize the estimation error and HYDRA-sketch memory footprint (marked with red stars). The orange diamond configuration is the suggested configuration based on the configuration strategies discussed in §4. Thus, our strategies result in a configuration comparable to the optimal configurations. This observation holds across all summary statistics and datasets.

Analysis of Performance Optimizations: Figure 15 depicts the cumulative improvement in HYDRA’s performance using the performance optimizations of §5. Each datapoint corresponds to a different HYDRA-sketch configuration (the Pareto frontier of Figure 14) and we run each configuration twice, once for the basic HYDRA-sketch design and once with the performance optimizations. The performance optimizations further reduce the memory footprint of HYDRA-sketch and the total system runtime. Table 2 captures HYDRA’s runtime reduction after each performance optimization. The baseline is HYDRA without optimizations; overall, we see a total performance improvement of 45%.

Skewness of Dataset: Figure 16 highlights the difference in estimation accuracy for two synthetic datasets generated with a zipfian distribution. The subpopulations are samples from a zipfian distribution. The subpopulations are samples from a zipfian distribution with parameters $\alpha = 0.7$ and $\alpha = 0.99$ respectively (a value of $\alpha = 0$ indicates a perfectly uniform distribution). Our experiment confirms our intuition that the more skewed dataset ensures a better (memory, error) tradeoff. In practice, many real-world datasets are skewed and thus can benefit from being analyzed by HYDRA.

7 RELATED WORK

MapReduce-based Analytics Frameworks: There are various analytics frameworks that are based on the MapReduce paradigm [48, 82]. Dryad [58] introduced the concept of user-defined functions in general DAG-based workflows. Apache Drill and Impula [65] are limited to SQL variants. Apache Spark [96] leverages a DAG-based execution engine and treats unbounded computation as micro-batches. Apache Flink [32] enables pipelined streaming execution for batched and streaming data, offers exactly-one semantics and
supports out-of-order processing. **Hydra** could be built on top of Apache Flink.

**Stream Processing Frameworks:** This line of research focuses on the architecture of stream processing systems, answering questions about out-of-order data management, fault tolerance, high-availability, load management, elasticity etc. [5, 14, 15, 21, 23, 27, 32, 57, 63, 73]. Fragkoulis et al. analyze the state of the art of stream processing engines [45].

**High-dimensional Data Cubes:** Data cubes have been an integral part of online analytics frameworks and enable pre-computing and storing statistics for multidimensional aggregates so that queries can be answered on the fly. However, data cubes suffer from the same scalability challenges as **Hydra**. Prior works have focused on mechanisms to identify the most frequently queried subsets of the data cube and optimize operations that are performed only on a small subset of dimensions at a time [49, 53, 54, 66, 68].

**Data Aggregations:** The aggregation-based queries that we discussed in §2 appear in multiple streaming data systems [20, 26, 28, 41, 52, 80, 93] that motivate **Hydra**. Many of the above frameworks enable approximate analytics but do not fully satisfy operators’ requirements as outlined in §2.

**Sampling-based Approaches:** Multiple analytics frameworks use sampling to provide approximate estimations [18, 34, 75, 89], BlinkDB [20] builds stratified samples on its input to reduce query execution time given specific storage budgets. STRAT [35] also uses stratified sampling but instead builds a single sample. SciBORQ [83] builds biased samples based on past query results but cannot provide accuracy guarantees.

**Online Aggregation:** Online Aggregation frameworks [55, 67, 77] continuously refine approximate answers at runtime. In these frameworks, it is up to the user to determine whether the acceptable level of accuracy is reached, and to terminate estimation. Naturally, this approach is unsuitable for multidimensional telemetry that needs to estimate multiple statistics across subpopulations.

**Data Summaries:** Data “synopses” (e.g., wavelets, histograms, sketches, etc.) have been extensively used for data analytics [19, 31, 40, 50, 59, 69, 88, 90]. These data summaries can either be lossless or lossy, and they aim to provide efficiency for multidimensional analytics. However, these approaches are tailored to a narrow set of estimation tasks. Gan et al. develop a compact and efficiently mergeable quantile sketch for multidimensional data [47].

Several prior efforts explore nested sketches as a solution to the multidimensional distinct counting problem [38, 85, 86, 91]. The CountMin Fajolet-Martin (CM-FM) replaces each integer counter of count-min sketch with a distinct counting sketch [38]. The CM-FM, while making a step in the right direction for multidimensional analytics, is limited both in terms of the generality and accuracy guarantees it offers [85]. Prior work by Ting et al. also targets on cardinality estimation in multidimensional data [85, 86] but focuses on improving the sketch error bounds. Similar to **Hydra**, they observe that in distinct counting sketches, accuracy guarantees depend on the characteristics of the underlying data. Their key observation is that the distribution of errors in each counter can be empirically estimated from the sketch itself. By first estimating this distribution, count estimation becomes a statistical estimation and inference problem with a known error distribution. However, computing such error distributions, is computationally heavy in streaming settings as it involves computing maximum likelihood estimators.

## 8 DISCUSSION AND FUTURE WORK

**Hydra** ensures coverage across subpopulations and accuracy guarantees with good resource utilization for subpopulations whose \( G_i > G_{\text{min}} \). It is up to the operator to determine \( G_{\text{min}} \). We believe that this is more versatile than pre-selecting specific subpopulations for which accuracy guarantees should apply. Given a \( G_{\text{min}} \) threshold, **Hydra** self-selects the subset of important subpopulations.

**Hydra** opens up avenues for future work. For example, an open question is how to enable dynamic sketch reconfiguration given changing workloads or operator goals. Also, a more system-oriented avenue would involve investigating the applicability of **Hydra** in the context of in-band network telemetry as part of programmable network elements [74].

## 9 CONCLUSIONS

Today’s large-scale services require interactive estimates of different statistics across subpopulations of their multidimensional datasets. However, the combinatorial explosion of subpopulations complicates offering multidimensional analytics at a reasonable cost to the operator. We propose **Hydra**, a sketch-based framework that leverages **Hydra**-sketch to summarize data streams in sub-linear memory to the number of subpopulations. We show that **Hydra** is an order of magnitude more efficient than existing analytics engines while ensuring interactive estimation times.

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